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Nonlinear Pitch Autopilot Design with Local Linear System Analysis

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Abstract

This paper presents the design of nonlinear feedback linearized pitch missile autopilots with local linear system analysis. Pitch acceleration control of a tail controlled missile exhibits non-minimum phase zero characteristics due to the adverse force/moment relationships from the tail. While direct application of the output feedback linearization technique can result in unstable zero dynamics, its impact on the control system performance can be minimized by invoking various approximations. Since these approximations have been made in deriving the feedback linearization control, some of the robustness properties will be lost during the inverse transformation process. The local linear system analysis is used to investigate the robustness properties of the two approximated nonlinear pitch autopilots. Simulation results illustrating robustness and tracking performance are presented.

Keywords: *Global Output Feedback Linearization Control, Missile Pitch Acceleration Autopilot, Non-minimum Phase System, Local Linear System Analysis.*

1. Introduction

Autopilot design has been dominated by the requirement of superagility in the entire flight envelope of the missile [1, 2, 3]. Missile flight control systems have to be capable of delivering agile performance while considering nonlinearities and uncertainties in the model. Generally, autopilot design procedure is to linearize the missile dynamics at several equilibrium conditions indexed by the scheduling variables, such as angle of attack, Mach, and height first. Then, the feedback gains at various reference flight conditions are determined using conventional linear control theory and then scheduled by key variables. The stability and performance characteristics of the flight control system are checked in linear system analysis and nonlinear simulation. The design process is repeated until the design requirements are met. Although it is theoretically possible to achieve any desired performance by designing controllers at closely spaced reference conditions and scheduling the gains, enormous amount of design efforts is required. In addition, due to more severe performance requirements in missiles, conventional autopilot design approaches may not be satisfactory in the whole flight envelope.

Nonlinear control design methods capable of delivering high performance without the need for gain scheduling offer an alternative to the conventional linear design techniques. One of these design methods is to apply feedback linearization techniques to the nonlinear missile dynamics. Applying these techniques achieves consistent flight control system performance, irrespective of flight conditions. However, difficulties arise with the non-minimum phase characteristic of tail controlled missiles when normal acceleration is commanded [2]. When these techniques are applied to non-minimum phase systems, it can leave the zero dynamics unstable. Thus, when controlling the accelerations of missiles, we cannot directly apply the feedback linearization control techniques. Various approaches have been used to alleviate this problem for the design of a pitch autopilot. The singular perturbation technique together with a partial linearization is employed in [4]. A two-time scale separation technique as well as output approximation is used in [5]. Since these techniques introduce various approximations to eliminate non-minimum phase zero, some of the stability robustness properties will be lost. In order to check these robustness properties their local linear system analyses should be performed.

In this paper, two nonlinear pitch autopilots are designed using approximated feedback linearization techniques proposed in [4, 5]. Moreover, their local linear system analyses are performed in order to check the stability margins and time responses of the pitch acceleration control systems. The robustness properties of two autopilots are compared.

The remainder of the paper is organized as follows: Section (2) describes missile dynamics used in this paper. Section (3) presents two approximation methods of feedback linearization control techniques considering the non-minimum phase zero characteristics. Section (4) contains the local linear system analysis and nonlinear simulation results. Finally, some concluding remarks are given in Section (5).

2. Missile Model

The missile dynamics considered here are taken from [1]. These dynamics are representative of a missile travelling at Mach 3 at an altitude of 20,000 ft.

The nonlinear missile dynamics are as follows:

$$\dot{\alpha} = f\,\frac{\cos(\alpha / f)}{mV} F_z + q \tag{0.1}$$

$$\dot{q} = f\,\frac{M_y}{I_y} \tag{0.2}$$

where

α = angle of attack, deg
f = radians-to-degrees conversion, 180/π
m = mass, 13.98 slugs
V = speed, 3109.3 ft/s
F_z = normal force, $C_z QS$, lb
M_y = pitch moment, $C_m QSD$, ft · lb
I_y = pitch moment of inertia, 182.5 slug · ft^2
Q = dynamic pressure, 6132.8 lb · ft^2
S = reference area, 0.44 ft^2
D= reference diameter, 0.75 ft

The normal force and pitch moment aerodynamic coefficients are approximated by

$$C_z(\alpha,\delta_p) = C_{z0}(\alpha) + C_{z\delta}\cdot\delta_p \tag{0.3}$$

$$C_m(\alpha,\delta_p) = C_{m0}(\alpha) + C_{m\delta}\cdot\delta_p \tag{0.4}$$

where

$$C_{z0} = 0.000103\alpha^3 - 0.00945\alpha\,|\,\alpha\,| - 0.170\alpha \tag{0.5}$$

$$C_{m0} = 0.000215\alpha^3 - 0.0195\alpha\,|\,\alpha\,| + 0.051\alpha \tag{0.6}$$

$$C_{z\delta} = -0.034, \quad C_{m\delta} = -0.206 \tag{0.7}$$

δ_p =fin deflection, deg

These approximations are accurate for α in the range of $\pm$ 20 deg.

The missile tail fin actuators are modeled as the 1st or 2nd order transfer functions as

$$\frac{\delta_p}{\delta_c}(s) = \frac{1}{\tau_a s + 1} \tag{0.8}$$

$$\frac{\delta_p}{\delta_c}(s) = \frac{\varpi_n^2}{s^2 + 2\varsigma\omega_n s + \varpi_n^2} \tag{0.9}$$

where

δ_c = commanded fin deflection, deg
τ_a = 1st order actuator time constant, 1/150 s
ς =2nd order actuator damping ratio, 0.7
ω_n =2nd order actuator bandwidth, 125.6 rad/s (20Hz)

The pitch autopilot will be required to control the body's z-axis (pitch) acceleration normalized by gravity:

$$n_z = \frac{F_z}{mg} = \frac{QS}{mg} C_z(\alpha, \delta_p) \tag{0.10}$$

Where g is the acceleration of gravity.

The nonlinear state equations (0.1) and (0.2) are linearized about trim operating points (M_y =0) to form linear state-space equations of the form

$$\begin{bmatrix}\dot{\alpha}\\ \dot{q}\end{bmatrix} = \begin{bmatrix} Z_\alpha & 1\\ M_\alpha & 0\end{bmatrix}\begin{bmatrix}\alpha\\ q\end{bmatrix} + \begin{bmatrix} Z_\delta\\ M_\delta\end{bmatrix}\delta_p \tag{0.11}$$

$$\begin{bmatrix}\alpha\\ q\\ n_z\end{bmatrix} = \begin{bmatrix} 1 & 0\\ 0 & 1\\ C_1 & 0\end{bmatrix}\begin{bmatrix}\alpha\\ q\end{bmatrix} + \begin{bmatrix} 0\\ 0\\ C_2\end{bmatrix}\delta_p \tag{0.12}$$

where

$$\begin{aligned} Z_\alpha &= -\frac{\sin(\alpha/f)}{mV}F_z + f\,\frac{\cos(\alpha/f)}{mV}\frac{\partial F_z}{\partial\alpha}\\ M_\alpha &= \frac{f}{I_y}\frac{\partial M_y}{\partial\alpha}\\ Z_\delta &= f\,\frac{\cos(\alpha/f)}{mV}\frac{\partial F_z}{\partial\delta_p}\\ M_\delta &= \frac{f}{I_y}\frac{\partial M_y}{\partial\delta_p}\\ C_1 &= \frac{1}{mg}\frac{\partial F_z}{\partial\alpha}, \quad C_2 = \frac{1}{mg}\frac{\partial F_z}{\partial\delta_p}\end{aligned} \tag{0.13}$$

Now, the purpose of a nonlinear pitch autopilot is that the plant output n_z tracks the pitch acceleration command n_{zc} asymptotically as

$$\lim_{t\to\infty} |\, n_{zc} - n_z \,| = 0 \tag{0.14}$$

3. Feedback Linearization Control

Feedback linearization is first implicitly developed in the context of nonlinear decoupling, which is closely related to the invertibility of the system [6]. Unlike the Jacobian linearization approach, this approach only utilizes feedback and coordinates transformations to render the given system a linear input-output dynamics. Feedback linearization is accomplished by repeatedly differentiating the system outputs until the control variables appear explicitly on the right hand sides of the resulting differential equations. In the flight control area, feedback linearization method has been used for designing autopilots for high performance missiles.