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Proportional-Integral-Plus control of a class of nonlinear systems using exact and partial linearisaton by feedback

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Abstract

This paper considers State Dependent Parameter (SDP), Proportional-Integral-Plus (PIP) control of a wide class of nonlinear systems. Here, the system is modelled using the quasi-linear SDP model structure, in which the parameters are functionally dependent on other variables in the system. This formulation is then be used to design a PIP control law using linear system design strategies, such as pole assignment or suboptimal linear quadratic design. However, since not all feasible SDP model structures can be solved using the basic approach, the present paper develops a novel *partial linearization by feedback* method, that returns the closed-loop system to a controllable state. The development of the new approach is motivated by limitations in the more familiar *exact linearization via feedback* technique also considered in the paper, when the latter is applied to SDP models.

Keywords: *discrete-time nonlinear system; linearization by feedback; proportional-integral-plus (PIP) control; state dependent parameter (SDP) model.*

1. Introduction

Previous papers have introduced the linear Proportional-Integral-Plus (PIP) controller [1-2], in which Non-Minimal State Space (NMSS) models are formulated so that full state variable feedback control can be implemented directly from the measured input and output signals of the controlled process, without resort to the design and implementation of a deterministic state reconstructor or a stochastic Kalman filter. Such PIP control systems have been successfully employed in a wide range of practical applications, e.g. [3-4]. Typically, however, any inherent nonlinearity in the system have been accounted for in a rather *ad hoc* manner at the design stage, sometimes leading to reduced control performance when applied to particularly difficult, highly nonlinear systems.

One novel research area currently being investigated in order to improve PIP control in such cases, is based on the State Dependent Parameter (SDP) system identification methodology. Here, the nonlinear system is modelled using a quasi-linear model structure, in which the parameters are functionally dependent on other variables in the system [5]. In this manner, SDP models can provide a description of a widely applicable class of nonlinear systems that even includes chaotic processes and systems that have previously been modelled using a bilinear approach.

The linear-like, 'affine' structure of the SDP model means that, at each sampling instant, it can be considered as a 'frozen' linear system. This formulation is then used to design an SDP-PIP control law using linear system design strategies such as pole assignment or suboptimal Linear Quadratic (LQ) design: see e.g. [6-7]. This yields SDP-PIP control systems in which the state feedback gains are themselves state dependent. However, not all feasible SDP model structures are controllable using this basic approach [8]. For this reason, the present paper develops a novel *partial linearization by feedback* method that allows the general SDP model form to be controlled. The new approach is motivated by limitations in conventional *exact linearization via feedback* methods (see e.g. [9]), when these are applied to SDP models.

2. Nonlinear SDP-PIP control

The representation of nonlinear dynamical systems using State Dependent Parameter (SDP) models goes back to Young [10]. However, the practical development of this model is of more recent origin: see [5] and the references therein. In this paper, we consider the following SDP model, written in discrete-time difference equation form,

$$\begin{aligned} y_k &= -f(y_{k-1})y_{k-1} - \cdots - f(y_{k-n})y_{k-n} \\ &\quad + f(u_{k-1})u_{k-1} + \cdots + f(u_{k-m})u_{k-m} \end{aligned} \tag{1}$$

where u_k and y_k are the input and output variables respectively. The parameters $f(y_{k-i})$ and $f(u_{k-i})$ are themselves functions of the lagged system variables. In Transfer Function (TF) form, the model (1) becomes,

$$y_k = \frac{b_{1,k} z^{-1} + \cdots b_{m,k} z^{-m}}{1 + a_{1,k} z^{-1} + \cdots a_{n,k} z^{-n}} u_k = \frac{B_k(z^{-1})}{A_k(z^{-1})} u_k \tag{2}$$

where z^{-1} is the backward shift operator, i.e. $z^{-i} y(k) = y(k-i)$. Equation (2) alludes to the time variable parameter derivation of the SDP model: see e.g. reference [5] for details of this identification step. Here, $a_{i,k} = f(y_k)$ for $1 \le i \le n$ and $b_{i,k} = f(u_k)$ for $1 \le i \le m$. By paying careful attention to the backward shift operator notation, it is clear that, for example, $a_{1,k}$ is a function of the un-lagged output variable y_k.

It is easy to show that the model (2) can be represented by the following Non-Minimal State Space (NMSS) form,

$$\begin{aligned} \mathbf{x}_{k+1} &= \mathbf{F}_k \mathbf{x}_k + \mathbf{g}_k u_k + \mathbf{d}\, r_{k+1} \\ y_k &= \mathbf{h}\, \mathbf{x}_k \end{aligned} \tag{3}$$

where $\mathbf{F}_k, \mathbf{g}_k$, $\mathbf{d}$ and $\mathbf{h}$ are defined by numerous earlier publications (for the time invariant case) e.g. [1-4]. The $n+m$ dimensional non-minimal state vector $\mathbf{x}_k$ consists of the present and past sampled values of the output and the past sampled values of the input variables, i.e.,

$$\mathbf{x}_k = \begin{bmatrix} y_k & y_{k-1} & \cdots & y_{k-n+1} & u_{k-1} & \cdots & u_{k-m+1} & z_k \end{bmatrix}^T \tag{4}$$

Here, $z_k = z_{k-1} + (r_k - y_k)$ is the integral-of-error between the reference and the sampled output. Inherent type 1 servomechanism performance is introduced by means of this state, z_k. The control law associated with the NMSS model (3) takes the usual State Variable Feedback (SVF) form,

$$u_k = -\mathbf{v}_k\, \mathbf{x}_k \tag{5}$$

where the state variable feedback gain vector is given by,

$$\mathbf{v}_k = [\, f_{o,k} \quad f_{1,k} \quad \cdots \quad f_{n-1,k} \quad g_{1,k} \quad \cdots \quad g_{m-1,k} \quad -k_{I,k} \,] \tag{6}$$

The vector $\mathbf{v}_k$ is itself state dependent, with control gains determined at each sampling instant by either pole assignment or optimisation in terms of a LQ cost function,

$$J = \frac{1}{2} \sum_{i=0}^{\infty} \left\{ \mathbf{x}_i^T \mathbf{Q} \mathbf{x}_i + R u_i^2 \right\} \tag{7}$$

where **Q** is a diagonal state weighting matrix and *R* is an additional scalar weight on the input [1-2].

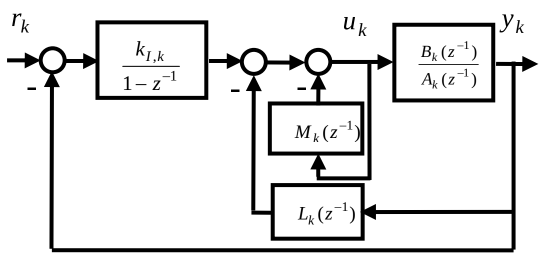

Figure 1. Conventional SDP-PIP control block diagram.

In more conventional block diagram terms, equation (5) can be implemented as shown in Figure 1, where $M_k(z^{-1})$ and $L_k(z^{-1})$ are defined as follows,

$$\begin{aligned} L_k(z^{-1}) &= f_{o,k} + f_{1,k} z^{-1} + \cdots + f_{n-1,k} z^{-1} \\ M_k(z^{-1}) &= g_{1,k} z^{-1} + \cdots + g_{m-1,k} z^{-1} \end{aligned} \tag{8}$$

3. Partial linearization by feedback

Consider, in the first instance, the ubiquitous continuous-time single-input, single-output nonlinear system,

$$\begin{aligned} \dot{x} &= f(x) + g(x) u \\ y &= h(x) \end{aligned} \tag{9}$$

Equation (9) is defined by Isidori [9] to have a relative degree *r* at a point x^o if,

$L_g L_f^k h(x) = 0$ for all *x* and all $k < r-1$

$L_g L_f^{r-1} h(x^o) \neq 0$

Here, $L_f h(x)$ is the derivative of the function *h(x)* along the function *f(x)*. Note that if $L_g L_f^k h(x) = 0$ for all *x*, then the relative degree cannot be defined, implying that the system output is not affected by the input, i.e. the output depends only on the initial states. For linear systems, the relative degree *r* is equal to the difference between the degree of the denominator and numerator polynomials of the transfer function.

For brevity, it is sufficient to note here, only that equivalent results can be developed for discrete-time systems, utilising $\Delta y_k = y_k - y_{k-1}$: see e.g. [17-19]. In particular, for the nonlinear SDP model (2), it is apparent that the relative degree is the number of samples required for the input to affect the system following a change in the equilibrium conditions; in other words, the sampled time delay δ. Such time delays are straightforwardly introduced into equations (1) and (2) by setting the leading 'input' parameters to zero. However, it is more convenient here to utilise the following revised model, in which the time delay $\delta \ge 1$ is explicitly acknowledged,

$$\begin{aligned} y_k =& -f(y_{k-1}) y_{k-1} - \cdots - f(y_{k-n}) y_{k-n} \\ &+ f(u_{k-\delta}) u_{k-\delta} + \cdots + f(u_{k-m}) u_{k-m} \end{aligned} \tag{10}$$

Proposition 1

For the nonlinear system (2), it is possible to find a linear input term with the same relative degree that approximates the nonlinear input terms, i.e.,

$$BU_{k-\delta} \cong f(u_{k-\delta}) u_{k-\delta} + \cdots + f(u_{k-m}) u_{k-m} \tag{11}$$

for which *B* is constant. In theoretical terms, when there is no model mismatch, *B* can be any scalar value. However, in the case of practical implementation, the robustness of the controller will be dependent on *B*. For this reason, Proposition 2 below considers the most appropriate nominal value of *B*.