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Tracking control of robots using Global H infinity robust control

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Abstract— Recent success in nonlinear $H\infty$ control design has been applied to control of robot manipulator systems. In this paper a Global $H\infty$ robust controller is considered for tracking control problem of Robotic manipulators. The synthesis of this controller is based on Lyapunov function instead of solving the HJI equation. Performance issues of the Global $H\infty$ robust controller are illustrated in a simulation study made for the two degrees of freedom manipulator controller. Comparison of its performance with the conventional controller of feedback linearization is also made for disturbance attenuation properties for different types of disturbances such as friction and bounded disturbances

*Keywords: **tracking**, control, robot, $H\infty$, friction, disturbances*

I. INTRODUCTION

In the past the development of robust control strategies for robot manipulator has received considerable interest [1]. Robust control strategies are designed to yield good dynamic behavior when confronted with modeling errors and unmodeled dynamics, as robot posses complex nonlinear, coupled dynamics. Model uncertainties are there due to changing payload, friction, backlash and flexibility in joints and links.

The approach of $H\infty$ control has been widely discussed for robustness and its capability of disturbance attenuation in robot control systems . $H\infty$ control involves two issues, one to stabilize the system and other to ensure that the L2 gain, from disturbance input to the controlled output, of the closed loop system is not larger than a prescribed value. So $H\infty$ controllers are optimal in the sense that, they are based on minimization of the cost function i.e.L2 gain.[2,3]

It is well known that design problem of nonlinear $H\infty$ optimal control reduces to solving a partial differential inequality or equation, called Hamilto-Jacobi-Isaccs (HJI), which is, in many cases impossible to solve explicitly. Therfore, much research aimed at solving nonlinear $H\infty$ optimal control in simpler form was proposed e.g., Battilotti and Lanari [4]solved disturbance attenuation problem indirectly based on the back stepping method ,and Nakyama and Arimoto [5] proposed a method of tuning .passivity based control to endow a sort of $H\infty$ optimality .Chen et al. [6] proposed one of the most successful design of nonlinear $H\infty$ optimal control. Applying the special coordinate transformation for quadratic optimal control of robot manipulators, it was shown that solutions of the associated HJI equation can be found by solving an algebraic matrix equation [6] but involves too much computational complexity..

Due to difficulties in solving HJI equation, many simpler solutions for $H\infty$ problems for nonlinear systems are concentrated to solve the problem locally. This local solution transforms the HJI equation into an algebraic Ricatti equation but they do not solve the problem globally. Su et. al [8] designed a nonlinear $H\infty$ controller for nonlinear time invariant system which ensures the global stability. Their design does not rely on solution of any HJIE's but instead a Lyapunov function is used to solve the $H\infty$ control problem, which also ensures global stability. An attempt has been made in this paper to implement this Lyapunov function based global robust $H\infty$ controller for the tracking control of robot manipulator.

High performance tracking control of robots cannot be achieved if friction phenomena are not properly taken into account[9]. To incorporate changing friction characteristics with velocity and for designing efficient friction compensation techniques, three empirical dynamic models are usually applied to the robot dynamic model [10,11]. They are static model, the Dahl model, and the LuGre model

The paper is organized as- Section 2 presents Robot dynamics and the different forms of friction and their models. Conventional controller of feedback linearization method known as computed torque controller and Robust $H\infty$ controller and their implementation for tracking control problem are given in section 3 and 4. Simulation of tracking control problem of Robot Manipulators with different frictions acting on it along with bounded disturbances is dealt in section 4. Finally section 5 gives the conclusion.

2. SYSTEM MODEL

2.1 Robot Dynamics The dynamics of revolute joint type of robot can be described by following nonlinear differential equation [12]

$$T = M(q)\ddot{q} + C(q,\dot{q})\dot{q} + G(q) + w \qquad (1)$$

Where $w = F(\dot{q}) + T_d$

With $q \in R^{ni}$ is the join position variables $\in R^n$, T is vector of input torques, M(q) is the inertia matrix which is symmetric and positive definite, $C(q,\dot{q})\dot{q}$ is the coriolis and centripetal forces, G (q) includes the gravitational forces ,w is the vector of perturbations. $F(\dot{q})$ represents dynamic friction forces and static friction forces acting independently in each joints and T_d is vector of bounded disturbances.

2.2 Friction model.

The classical model for friction involves incorporating coulomb and viscous friction models. The following static model of friction is considered in this paper:[10]

$$F = F_v \dot{q} + F_c \operatorname{sgn}(\dot{q}) \quad \text{-------------(2)}$$

Where F_v is a diagonal matrix with diagonal elements as static friction constants, similarly F_c is a diagonal matrix representing coulomb friction constants, $\dot{q}$ is joint angle velocity for each link of the manipulator

As opposed to classical static friction model, dynamic friction models attempt to incorporate a variety of other friction characteristics such as stiction, zero slip displacement, stribeck effect etc. Dynamic friction models also tend to capture effectively the changing friction characteristics that are caused primarily due to wear and aging. The early dynamic friction model was given by Dahl model [9] in the sliding regime by . (3)

$$\dot{F}_d = \sigma_0 \dot{q} - \sigma_0 |\dot{q}| \frac{F_d}{F_c} \quad \text{-----(3)}$$

Where σ_0=stiffness coefficient,

General friction model is given by Eq. (4) as a superposition of viscous friction and Dahl friction

$F = \sigma_1 \dot{q} + F_d$ -------------(4)

where σ_1 is viscous friction coefficient.

Dahl's model behaves like a spring for small deflection q and approaches coulomb friction for large deflection. Dahl's model account for coulomb friction but it does not include the stribeck and stiction effects.

Recently a new friction model named LuGre model was proposed which is an extension of Dahl model and includes many aspects of dynamic friction. This LuGre friction model supports hysterisis behavior due to frictional lag, spring like behavior in stiction and gives a varying break away force depending on the rate of change of applied force, and was proposed by Canudas de wit et.al [11]. All these phenomena are unified into a first order nonlinear differential equation, given by

$$F = \sigma_0 z + \sigma_2 \frac{dz}{dt} + \sigma_1 \dot{q} \quad \text{---(5)}$$

where

$$\sigma_0 g(\dot{q}) = F_c + (F_s - F_c)e^{-(\dot{q}/v_s)^2}$$

and $\frac{dz}{dt} = \dot{q} - \frac{|\dot{q}|}{g(\dot{q})} z$

Where σ_0 is stiffness coefficient, $\sigma 1$ is viscous friction coefficient, σ_2 is the damping coefficient, z is the state variable, g ($\dot{q}$) is the friction that models the constant velocity behavior which describes the stribeck effect, v_s is the stribeck velocity, F_c and F_s are coulomb and static friction level.

3. Tracking control of robot manipulators

In the motion control of mechanical manipulator, it is essential for the manipulator to follow the desired trajectories as close as possible at fast operational speed. So the primary concern of controller is the tracking of the coordinates q to some desired trajectory qd (t). The position error is defined as

$$e = qd - q \quad \text{-----(6)}$$

Using this transformed coordinates, the following well-known control [12], termed as computed torque control is considered,

$$M(q)(\ddot{q}^d + K_v\dot{e} + K_p e) + C(q,\dot{q})\dot{q} + G(q) = T \quad \text{----(7)}$$

Where K_v and K_p are symmetric positive definite matrices. The closed loop systems given by eq. (1) and,eq. (7) is globally asymptotically stable when w=0 i.e all the disturbances, friction and uncertainties are not present [12]

But in practical situations w is never be equal to zero. There will be many different types of disturbances acting on the robots such as frictions, parameter variations, change in payload and other nonlinearity effects. Under the influence of these inputs performance and stability can not be guaranteed with the conventional controllers such as computed torque given by eq.(7).Hence the need arises for the robust controller which can take into account these disturbances also.

4. Global H∞ tracking controller

To compensate the effect of friction, disturbances and model uncertainty a new auxiliary control input u which is a solution of nonlinear H_∞ control problem [13] given by the Eq. (8) is taken. This controller is robust in the sense of l_2 gain (γ) attenuation from external disturbances as well as due to model uncertainties

$$u = -\frac{1}{2}\begin{bmatrix} 0_{n\times m} \\ -M(q)^{-1} \end{bmatrix}^T \left(\frac{\partial V}{\partial X}\right)^T \quad \text{-----------(8)}$$

where V is suitable Lyapunov candidate and X is the state vector, with the condition that required properties for the existence and uniqueness of solution are satisfied .

The conventional tracking controller given by (7) can be made robust by adding the new auxiliary H∞ control u given by (8). So the global H∞ tracking controller of robot manipulator is given by the following eq.(9)

$$M(q)[\ddot{q}^d + k_v\dot{e} + k_p e) + V(q,\dot{q})\dot{q} + G(q) + u = T$$

--- (9)

By putting (9) in (1), the following generalized form of closed loop system is obtained

$$\frac{d}{dt}\underbrace{\begin{bmatrix} e \\ \dot{e} \end{bmatrix}}_{x} = \underbrace{\underbrace{\begin{bmatrix} 0_{n\times n} & I_n \\ -K_p & -K_v \end{bmatrix}}_{A}\underbrace{\begin{bmatrix} e \\ \dot{e} \end{bmatrix}}_{x}}_{f(x,t)} + \underbrace{\begin{bmatrix} O_{n\times n} \\ -M(q)^{-1} \end{bmatrix}}_{g_2(x,t)} u + \underbrace{\begin{bmatrix} O_{n\times r} \\ -M(q)^{-1} \end{bmatrix}}_{g_1(x,t)} w$$

The output to be controlled is defined as follows

$$Z = \underbrace{\begin{bmatrix} I_n & O_{n\times n} \\ O_{n\times n} & I_n \\ O_{m\times n} & O_{m\times n} \end{bmatrix}}_{h1(x,t)}\begin{bmatrix} e \\ \dot{e} \end{bmatrix} + \underbrace{\begin{bmatrix} O_{n\times m} \\ O_{n\times m} \\ I_m \end{bmatrix}}_{k12(x,t)} u \qquad (11)$$

For the system given above it is required to find the solution of the Lyapunov equation $PA+A^TP= -Q$, for a given Q >0,to find suitable Lyapunov candidate $V(x)= X^TPX$ where $X = [e, \dot{e}]$ .The existence of V(x) is guaranteed due to $K_v = K_v^T > 0$,and $K_p = K_p^T > 0$.

4. Simulation

The problem of tracking control of a planar robotic arm having two degrees of freedom, having both the joints as revolute is taken up in this simulation study. The dynamics of the manipulator satisfies Eq. (1) and is affected by various friction models as given in section 2.2, and bounded disturbances.

A two link manipulator with the following system parameters is considered -link masses = m_1, m_2 =10 kg; link length = l_1, l_2 =. 4m; joint angles = q_1, q_2 (rad), applied torques=T_1, T_2(Nm). The dynamic equation matrices for two-link manipulator are of following form [12]

$$M(q)=\begin{bmatrix} (m_1+m_2)l_1^2 & m_2l_1l_2(S_1S_2+C_1C_2) \\ m_2l_1l_2(C_1C_2+S_1S_2) & m_2\ 12^2 \end{bmatrix}$$

$$C(q,\dot{q}) = m_2l_2l_1\begin{bmatrix} q_2 & 0 \\ 0 & q_2 \end{bmatrix},$$

$$g(q) = \begin{bmatrix} -(m_1+m_2)l_1gS_1 \\ -m_2l_2gS_2 \end{bmatrix}$$

----------------(12)

Short hand notations C_1=cos (q_1) and S_1=sin (q_1) etc. are used. Manipulator is commanded to trace a trajectory given by cubic polynomial with initial joint angle configuration as $q_1(0) = 10^o$, $q_2(0)= 15^o$ and final joint angle configuration $q_1(t_f) = 75^o$ and $q_2(t_f) = 65^o$ in t_f=1 second [5] shown in figs (1) and (2).

Friction parameters taken from the literature are as following.

$$(10)\ \sigma_0 = \begin{bmatrix} 1 & 0 \\ 0 & 1.2 \end{bmatrix}, \quad \sigma_1, \sigma_2 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix},$$

$$F_c = \begin{bmatrix} 5 & 0 \\ 0 & 1 \end{bmatrix},$$

$$F_s = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, \upsilon_s = \begin{bmatrix} 0.01 & 0 \\ 0 & 0.01 \end{bmatrix}$$

And T_d=[-10; 20] as bounded disturbance torque.

Case1) tracking performance with conventional controller For conventional computed torque controller of eq.(7) with u=0, gain K_p and K_v are tuned using set algorithm as 100 and 20 respectively for w=0 .At these optimal values the tracking error is minimum. The performance of this controller when applied for tracking control of robot manipulator is shown in figs. (3-6). The tracking errors in both joints have been shown in fig (3-4) and velocity error is shown in fig (5-6). This clearly indicates that performance of the conventional controller deteriorates quite much when friction and fixed disturbances (i.e.w) are introduced in the system model with the tuned parameters remaining the same .

Case2) with Robust Global H∞ controller

With the same simulation set up as above, robust controller of (9) is applied on robot manipulator for tracking control. K_p and K_v are selected as 100 and 20 respectively same as above. The solution to the Lyapunov equation with Q=diag (100) obtained was

$$P = \begin{bmatrix} 261.67 & 0 & .33 & 0 \\ 0 & 261.67 & 0 & .3333 \\ .3333 & 0 & 1.6778 & 0 \\ 0 & .3333 & 0 & 1.6778 \end{bmatrix}$$

Static and dynamic friction models given in section 2.2 and fixed disturbance torque T_d is also applied to the dynamic equation of the robotic manipulator. The purpose of this study is to investigate the performance of controller under different friction models and fixed disturbances. The tracking errors with this controller for different cases are shown in figs (7-8)- and velocity errors are shown in figs (9-10). Although there is slight increase in errors when permanent input disturbance torques and friction of different type is applied but the effects are insignificant and performance remains approximately the same. These figures additionally demonstrates robustness of this controller against friction discrepancy

Comparison of the results of the above two cases is shown in table1 which clearly indicates that performance deteriorates drastically in case 1(with conventional controller) particularly in joint angle 2 when fixed disturbance torque and friction is applied whereas in case of Robust Global H∞ controller it remains approximately the same.

Table1-Tracking error of joint angle 1and 2

Tracking error for above two cases		Absolute value of max. Tracking error		2 norm of Tracking error	
		Joint angle 1	Joint angle 2	joint angle1	Joint angle 2
Case1) Fig(3,4)	Without friction	0.147	0.6367	2.52	10.5
	With coloumb friction	0.235	2.3822	4.55	51.04
	With Dahl friction	0.140	0.785	2.40	16.18
	With LuGre friction	0.09	0.15	2.45	26.9
Case2) fig (7,8)	With out friction	0.066	0.0748	1.15	1.539
	With coulomb friction	0.107	0.1604	2.04	2.272
	With Dahl friction	0.084	0.1413	1.616	3.2694
	With LuGre friction	0.086	0.152	2.515	4.1496

5. Conclusion

A Global Robust H∞ controller is implemented for accurate trajectory tracking, which guarantees robustness to different frictions and disturbances. Extensive simulation studies have been carried out under a variety of conditions, which have demonstrated robustness, and tracking performance of the Robust H∞ control scheme. To facilitate exposition, the friction model chosen for treatment has been static friction and dynamic friction model, to carry out useful robustness properties of the controller against different friction forces. Comparative study of its performance with conventional computed torque control shows its superior robustness and performance. The drawback of this controller is that it depends upon the model parameters such as inertia parameters. The work can be extended to compensate for model parameters and gravitational effects.

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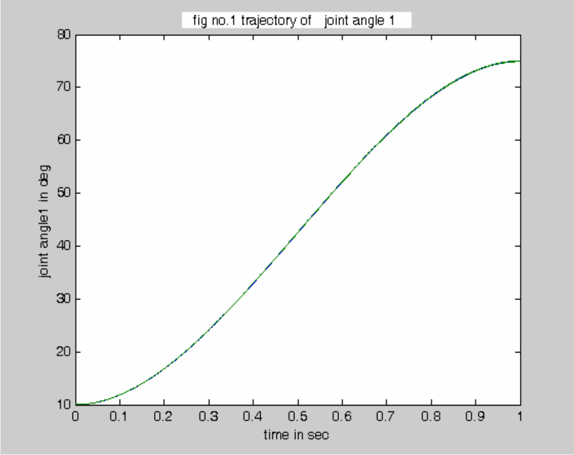

Fig.1 Trajectory of joint angle 1

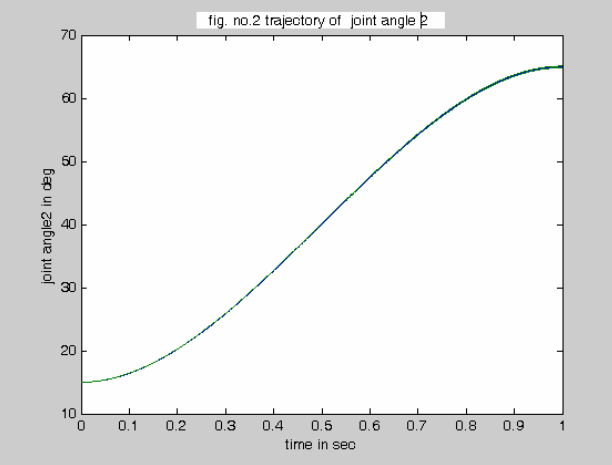

Fig. 2 Trajectory of joint angle2

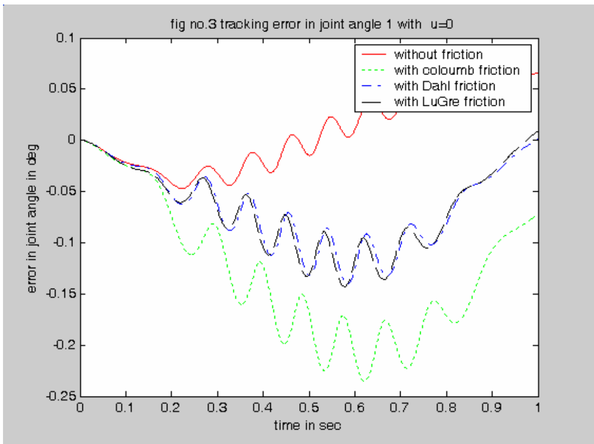

Fig.3 Tracking error in joint angle 1 with u=0

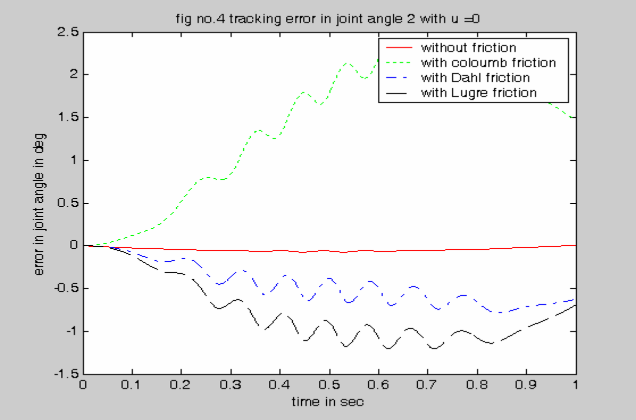

Fig. 4 Tracking error in joint angle 2 with u=0

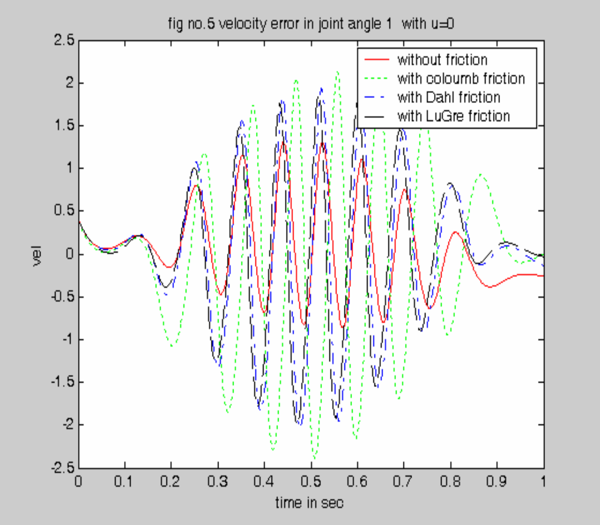

Fig .5 velocity error in joint angle 1 with u=0

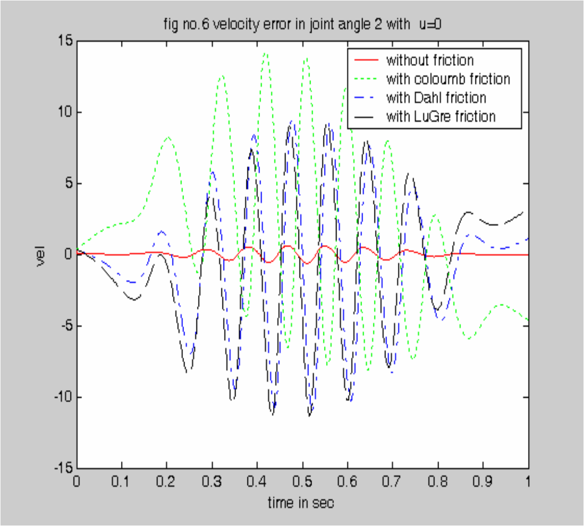

Fig..6 velocity error in joint angle 2 with u=0

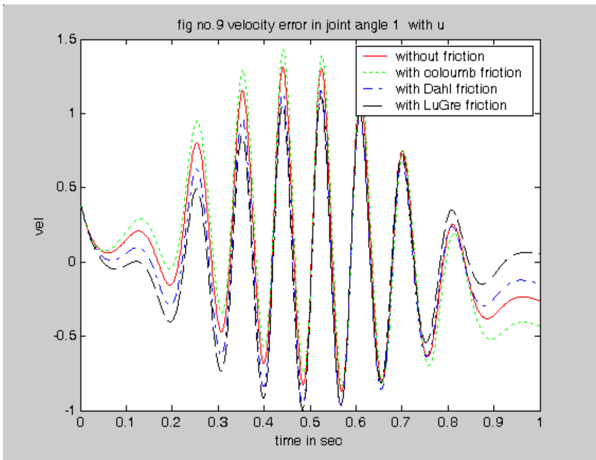

Fig.9 Velocity error in joint angle 1 with H∞ controller

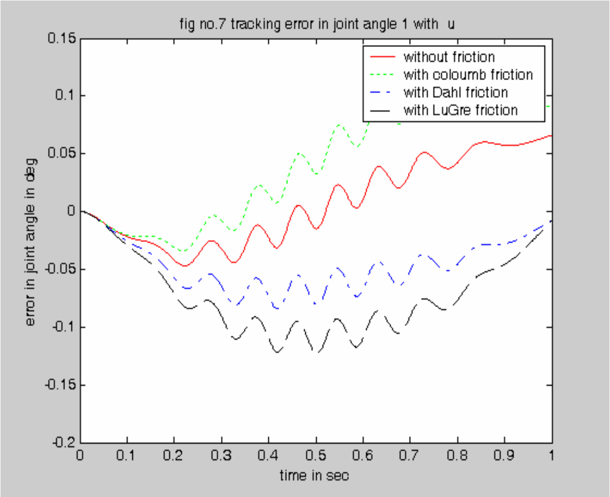

Fig. 7 Tracking error in joint angle 1 with H∞ controller

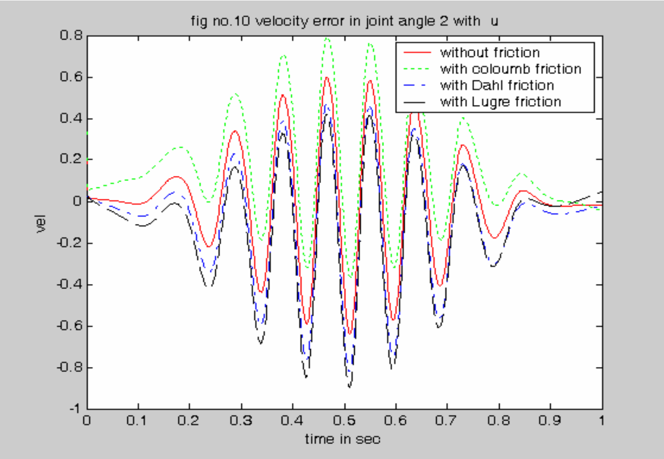

Fig10 Velocity error in joint angle 2 with H∞ controller

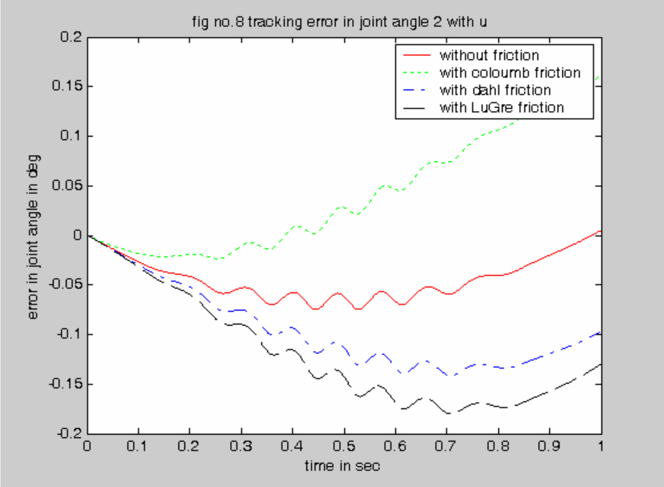

Fig. 8 Tracking error in joint angle 2 with H∞ controller