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A New Unsupervised Color Image Segmentation Algorithm upon a Statistical Multidimensional Data Analysis Approach

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Abstract

The problem of segmenting images into coherent regions has been a major subject of research in the field of computer vision. A new unsupervised color image segmentation algorithm which is use the hyperbolic filter for detecting modal regions of a multivariate probability density function is presented in this paper. The algorithm is carried out in a four stages processing, where the convexity dependant a separable recursive hyperbolic filter suitable for mode detection in segmenting analysis. It starts by estimation of the density function, followed by convexity dependant the filtering process, mode extraction to connected components and classification process. The algorithm has proven to be effective under a number of real and synthetic test images.

Keyword: *Hyperbolic operator, mode detection, data analysis, color image segmentation.*

1. Introduction

Color image segmentation consists in partitioning the image into disjoint regions as sets of connected pixels that share uniform color characteristics [1]. The approaches to the segmentation of color image can be classified into four groups, namely histogram-based techniques, neighborhood - based segmentation, physically - based segmentation techniques, and multidimensional data classification methods. In these approaches, most algorithms treated are based on threshold selection or in parameters adjustment which may change the segmentation results.

For the multidimensional data analysis methods [2, 3], color image segmentation is achieved by pixel classification according to color features. The clustering problem can be formally stated as follows. Considering Q pixels of the image represented as Q data points scattered through a color metric space, determine a partition of these Q points into C groups such that points within a group are more similar than points in different groups. In many situations, the number C of groups is not a priori known and has also to be determined. The statistical approach in data analysis postulates that the input patterns are drawn from an underlying probability density function (pdf) which describes the distribution of data points in the data space. Regions of high local densities, which might correspond to significant groups in the population, can be found from the modes of the density function estimated from available patterns [4]. Then, the key problem is to partition the data space with a multimodal pdf into subspaces over which the pdf is unimodal [5].

Given the analogy between segmentation and clustering, some image segmentation procedures, based on the grey level function analysis, have been successfully adapted to multidimensional probability density function analysis for pattern classification purpose. In this context some approaches based on relaxation [6], on edge detection [7], on mathematical morphology [8, 9] or on neural networks [10] have been proposed as efficient tools for multidimensional data analysis by means of mode detection procedures.

Among most of these existing procedures for mode detection of the underlying pdf, preliminary to unsupervised statistical data analysis, the ones that research modes as regions where the pdf is concave remain very interesting approaches [11, 12, 13]. These techniques make use of a test that determines locally the convexity of the underlying pdf from the input patterns. However, the test area of sampling points may straddle a boundary between a convex region and a concave one, so that the assumptions for the test of the local convexity can be violated.

Instead the local convexity testing used in the previous techniques, the proposed technique is based on the characteristic theorem of convexity [14]. It assigns the concave label to modal regions and the convex label to valleys of the pdf according to the sign of the Laplacien of this density function. The key problem is then to choose the adequate operator for computing the Laplacien, which might be no very sensitive to irregularities of the function. Indeed, the proposed algorithm does not call for any smoothing procedure as a pre-processing step of the process understanding since it uses an operator with a reliability criterion allowing to model as well the pdf variations as the noise attached to the density function. This operator [15], called hyperbolic operator as its impulsional response is a damped hyperbolic sine function, has been up to now concerned with edge detection of noisy gray level images [15, 16, 17]. In the present work, we show that its extension to a N-dimensional space is trivial since it is a separable recursive filter, and we expose the different steps used to achieve data analysis based on the hyperbolic Laplacien of the pdf.

The performance of this algorithm is first demonstrated using a synthetic distribution of data. The technique is then applied for color image segmentation.

2. Hyperbolic Operator

The hyperbolic operator is a recursive filter with an infinite impulsional response which satisfies the two optimality criterions of edge detection and localization defined by Canny [18] and the Sarkar's criterion of multiple responses [19].

The impulsional response of this filter is a damped hyperbolic sine function such as :

$$f_{hyp}(x)=C_d e^{-\alpha|x|}\sinh(\beta x)=C_d e^{-\alpha|x|}\frac{e^{\beta x}-e^{-\beta x}}{2} \qquad (1)$$

α and β $0<\alpha<\beta$ are two parameters whose adjustment is pertinent for the behavior of this operator to give good edge detection.

C_d is a parameter which conditions the normalization of the operator such as the response of the filter to a unit scale is equal to 1 at an edge position. This constraint allows to :

$$C_d=\frac{2(1-e^{(\beta-\alpha)})(1-e^{-(\alpha+\beta)})}{e^{-(\alpha+\beta)}-e^{(\beta-\alpha)}} \qquad (2)$$

The second derivative of the hyperbolic filter is given by :

$$L_{hyp}(X)=\frac{\alpha+\beta}{2}e^{-(\alpha+\beta)|x|}+C_L\frac{\beta-\alpha}{2}e^{(\beta-\alpha)|x|} \qquad (3)$$

$$\text{with : } C_L=-\frac{(\alpha+\beta)(1+e^{(\beta-\alpha)}-e^{-(\alpha+\beta)}-e^{-2\alpha})}{(\alpha-\beta)(1-e^{(\beta-\alpha)}+e^{-(\alpha+\beta)}-e^{-2\alpha})} \qquad (4)$$

To eliminate the noise which is generally superposed to useful information, the impulsional response of the smoothing filter $s_{hyp}(x)$ is given by a simple integration of $f_{hyp}(x)$ such as :

$$s_{hyp}(x)=C_s\frac{e^{-(\alpha+\beta)|x|}}{2(\alpha+\beta)}+\frac{e^{(\beta-\alpha)|x|}}{2(\beta-\alpha)} \qquad (5)$$

C_s is the coefficient of normalization defined by :

$$C_s=-\frac{(\alpha^2-\beta^2)(1-e^{(\beta-\alpha)}-e^{-(\alpha+\beta)}+e^{-2\alpha})}{\beta(1-e^{-2\alpha})+\alpha(e^{(\beta-\alpha)}-e^{-(\alpha+\beta)})} \qquad (6)$$

In the present study, we try to apply this filter in a N-dimensional space in order to use it as a tool for detecting modal regions of a multivariate pdf.

Let $p(x_1,...,x_N)$ be a N-dimensional function, $S_N(x_1,....,x_N)$ the studied N-dimensional hyperbolic smoothing filter and $p_s(x_1,...,x_N)$ the smoothed version of $p(x_1,...,x_N)$ by this filter. Let us consider the impulsional response of the N-dimensional filter $S_N(x_1,....,x_N)$ as a combination of N mono-dimensional hyperbolic smoothing filters such as :

$$S_N(x_1,....,x_N)=\prod_{i=1}^{N}s_{hyp}(x_i) \qquad (7)$$

According to the separability property of such filter, $p_s(x_1,...,x_N)$ can be obtained by processing separately and sequentially N convolutions such as :

$$p_s(x_1,...,x_N)=p(x_1,...,x_N)*\prod_{i=1}^{N}s_{hyp}(x_i) \qquad (8)$$

The Laplacien of this smoothed function is :

$$L_{p_s}(x_1,...,x_N)=\sum_{i=1}^{N}\frac{\partial^2 p_s}{\partial x_i^2}(x_1,....,x_N) \qquad (9)$$

The computation of this Laplacien necessitates at first to calculate the partial derivatives of p_s in each direction x_i, $i=1,2,..., N$, such as :

$$\frac{\partial p_s}{\partial x_i}(x_1,...., x_N)=p*\frac{\partial S_N}{\partial x_i}(x_1,...., x_N) \qquad (10)$$

with (cf. equations 1, 5 & 7) :

$$\frac{\partial S_N}{\partial x_i}(x_1,....,x_N)=f_{hyp}(x_i)\prod_{j=1,j\neq i}^{N}s_{hyp}(x_j) \qquad (11)$$

Let us denote by $D_i(x_1,....,x_N)$ the second derivative of p_s in the direction x_i. Thus :

$$D_i(x_1,...,x_N)=\frac{\partial^2 p_s}{\partial x_i^2}(x_1,...,x_N)=p*\frac{\partial^2 S_N}{\partial x_i^2}(x_1,...,x_N) \qquad (12)$$

with (cf. equations 3, 5 & 7) :

$$\frac{\partial^2 S_N}{\partial x_i^2}(x_1,....x_N)=L_{hyp}(x_i)\prod_{j=1,j\neq i}^{N}s_{hyp}(x_j) \qquad (13)$$

To be more specific, $D_i(x_1,....,x_N)$ is computed as:

$$[p(x_1,...,x_N)*s_{hyp}(x_1)*...*s_{hyp}(x_{i-1})*s_{hyp}(x_{i+1})*...*s_{hyp}(x_N)]*L_{hyp}(x_i) \qquad (14)$$

According to the equation (14), the directional second derivative of the smoothed function p_s is the function D_i computed in the direction x_i by convolution of $p(x_1,...,x_N)$ sequentially with successive impulsional responses of the smoothing filter $s_{hyp}(x_j)$ in the (N-1) directions $x_j\neq x_i$; the result of this sequence of convolutions is finally convoluted with $L_{hyp}(x_i)$, which is the second derivative in the direction i of the hyperbolic filter (cf. equation 3).

The Laplacien of p_s is then given as:

$$L_{p_s}(x_1,...,x_N)=\sum_{i=1}^{N} D_i(x_1,...,x_N) \qquad (15)$$

3. Data Analysis Algorithm

When modes are detected by the local test of convexity of a discrete pdf, the local convexity at a sampling point is determined by analyzing the variations of the mean value of the pdf computed within a family of domains expanding around the current sampling point [11, 12, 13]. Due to the presence of sparse data points scattered around the modes and to the presence of holes in the distribution of the data points within the groups, the modes are sometimes rather ill defined by their local convexity properties.

The technique for mode detection proposed here postulates that modal regions of the pdf correspond to those where the Laplacien of the pdf is found negative. However, in practice the shape of the underlying pdf is far from the ideal model with strictly concave modal regions, separated by well-defined convex valleys. The choice of the Laplacien operator which might be no very sensitive to irregularities of the pdf has then a major effect of the issue of the assumption for the analysis of convexity by the sign of the Laplacien function. Having this problem in mind, we have chosen the hyperbolic Laplacien which, in addition to its being a filtering process, depends on the two parameters α and β (cf. equation 3) which govern the behavior of the procedure. Furthermore, an adequate adjustment of these parameters leads to a good localization of the modal regions of the pdf.

A data analysis algorithm, which takes advantage of this filtering scheme, is presented using a two-dimensional data set. The data for this example, which are shown in figure 1, are drawn from three normal populations composed of 400 observations each.

The use of artificially generated two-dimensional data set aims to make the results easy to display and interpret, and to valid the proposed data analysis algorithm. Indeed, constructed sets are well adapted to validation studies since the true data structure is known, allowing an objective evaluation of the performance of the algorithm.

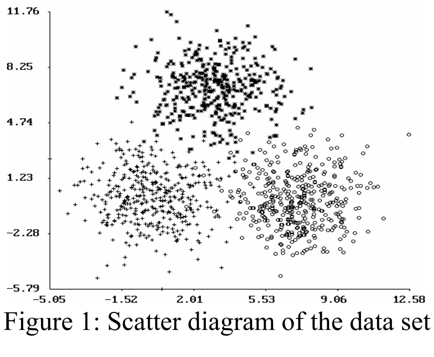

Figure 1: Scatter diagram of the data set

Step 1 : Estimation of the Density Function

To determine the uniform kernel estimate of p(X) from the set $\Gamma=\{X_1, X_2, ..., X_q, ..., X_Q\}$ of the Q available observations: $X_q=[x_{q,1},x_{q,2},...,x_{q,n},..,x_{q,N}]^T$, we use a fast non-parametric algorithm [20]. This one consists at first to normalize the range of variation of each component of the multivariate observations to an interval [0,K], where K is an integer. Each axis of this space is then partitioned into K exclusive and adjacent intervals of unit width. This discretization defines a set of K^N hypercubes of unit side length. The centers of these hypercubes constitute a regular lattice of sampling points denoted X. The underlying pdf is then estimated only at the centers of the non-empty hypercubes. All the available information for clustering is then the set $S \subseteq Z^{+N}$ of hypercubes where the sampled density function is defined not null. Figure 2 schematizes the graph of the uniform kernel estimate of the underlying pdf p(X) from patterns of figure 1, obtained with K=26. The choice of the value of the K will be justified in the § 4.

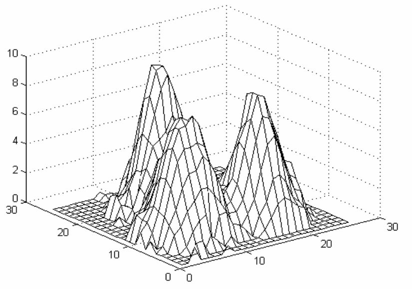

Figure 2: Raw estimate of the underlying pdf

Step 2 : Convexity dependant the filtering process

This principal step of the procedure consists at the localization of modal regions of the underlying pdf by means of the Hyperbolic Laplacien Algorithm (HLA) which is also a filtering process. The result L_{p_s} of the HLA applied with $\alpha=2\beta=2$ to the raw estimate p(X) of figure 2 is shown in figure 3. The choice of α will be discussed in § 4.

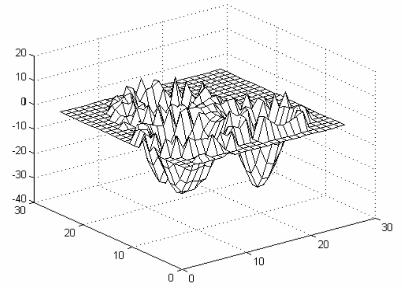

Figure 3: hyperbolic Laplacien function (hLf)

Step 3 : Mode Extraction to connected components

An assignment of each sampling point into a “modal” or a “non modal” region is based on the analysis of the sign of the hyperbolic Laplacien function. Modal regions, denoted by the subset $M_p(X)$ are detected and identified such as :

$$\forall X \in \underline{S}, \begin{cases} \text{if} \quad L_{p_s}(X) < 0 \quad \text{then} \quad M_p(X) = -L_{p_s}(X) \\ \text{otherwise,} \quad M_p(X) = 0 \end{cases} \qquad (16)$$

To illustrate this process, let figure 4 which schematizes the inverse of the hyperbolic Laplacien function of figure 3. It is shown that the support in $\underline{S}$ of sampling points corresponding to regions where the row function of figure 2 is concave includes will the support in $\underline{S}$ of sampling points where the inverse Laplacien function of figure 4 is positive. Modal regions of p(X) can then be extracted to separated connected components in the set $\underline{S}$ simply by conserving in the inverse Laplacien function only sampling points where this function is found positive. Figure 5 shows the graph of the function M_p (cf. equation 16) where three connected modal regions, resulting from the described mode extraction process, are will separated.

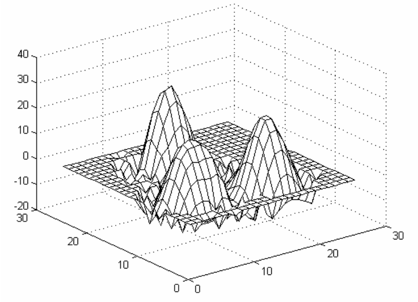

Figure 4: Inverse of the hLf

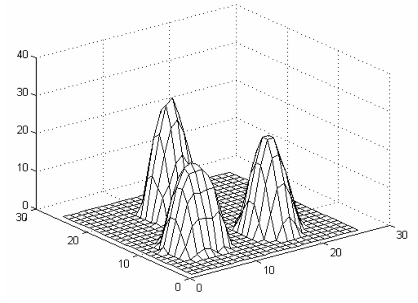

Figure 5: Extracted modal regions

Step 4 : Classification

Once the different modal regions of the underlying pdf are identified, the observations falling into them are used as prototypes. The remaining observations, which do not fall in one of the detected modal regions, can finally be assigned to the groups attached to their nearest neighbor among the prototypes [21]. The result, achieved by this process based on the extracted modal regions of figure 5, is shown in figure 6 which can be compared to figure 1.

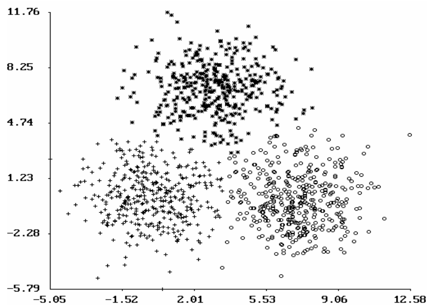

Figure 6: Result of clustering

The obtained error rate is equal to 2.66% for the proposed approach ; whereas it is equal to 3% with the Zhang mode detection technique for data analysis [12], and to 2.75% either with the Isodata and the K-means [22] algorithms. The difference between the error rate obtained with the proposed algorithm and the 2.51% Bayes theoretical minimum error rate corresponds to only two data points misaffected out of a total of 1200. In this example, and on the basis of these measures of effectiveness, it appears that the proposed data analysis scheme yields good results for a non-supervised segmentation.

4. Tuning the algorithm

The behavior of the proposed procedure for mode detection depends to a large extent on the adjustment of the resolution parameter K of the discretization process and the parameters α and β of the hyperbolic operator.

With given relatively great values of α and β, if K is too small, the estimate of the pdf will suffer from a too low resolution and small clusters will not be detected by the procedure. If K is too large, several concave domains, with a low altitude and a small size, will be found in the graph of the pdf. Hence, the procedure will tend to find out a great number of non-significant clusters. On the other hand and with an adequate value of the resolution parameter K, if the parameters α and β are too small, the hyperbolic Laplacien cannot possibly separate the concave domains from the convex ones so as that only one mode will be detected by the procedure.

In fact, the adjustment of these parameters depends both on the sample size Q and the dimensionality N of the data. Furthermore, the structure of the distribution of the observations has also an influence on the selection of these parameters. When nothing is a priori known about this structure, it can be expected that, with α and β given, the true clusters of the distribution are detected for a wide range of values of K. Reciprocally, with a given K and fixed relation between α and β, the true clusters of the distribution are detected for a wide range of values of α. For tuning the value of β in relation with α, we have taken nine relations between α and β : $\beta = i\alpha/10$ for i = 1, 2,…, 9. The application of this process to many underlying density functions estimated from different data sets has shown that the relation $\beta = \alpha/2$ with a great value of α

leads to good results. It is the reason why this relation has been considered in all experiments presented in this paper.

Under these assumptions, the adjustment of these parameters can be governed by the concept of mode stability [23]. For each value of K, the mode detection procedure is applied with some values of α (with $\beta = \alpha / 2$) ; mode stability is then satisfied in the largest domain resulting from the intersection of the largest ranges of K and the largest ranges of α where the number of detected modes remains constant (cf. figure 7). Choosing the parameter K in the middle of the largest range of K belonging to this domain of mode stability and choosing the parameter α as the lowest value of the largest ranges of α which intersect the range of the chosen K in this domain has proved to be a good procedure to optimize the proposed mode detection approach.

Figure 7 shows the effects of the parameters K and α (with $\beta = \alpha / 2$) on the number of detected components for the presented example, used to illustrate the clustering scheme. The longest stable string of K in the domain of mode stability occurs between K=20 and K=32. It is the reason why K has been taken equal to 26, which is the middle of that string. α =2 is the lowest value of the largest ranges of α which intersect the range of K=26 in this domain. The choice of the lowest value of α in this domain yields to have the shapes of the resulting modal regions large enough so that they reflect well the shapes of the clusters in the data.

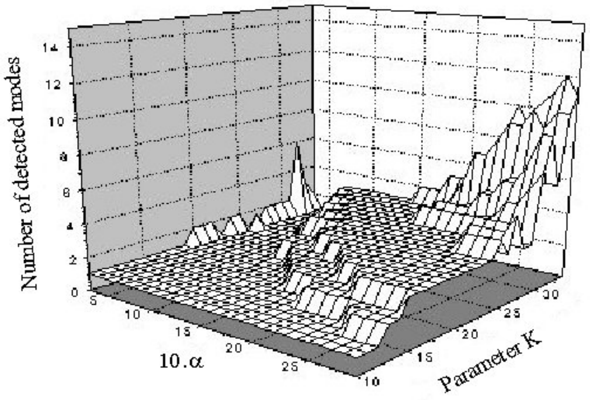

Figure 7: The effect of the resolution parameter K and the parameter α (with β=α/2) on the number of detected modal regions. of the presented example

In spite of the degree of overlapping between the clusters in the data set of this model example, the domain of mode stability, which represents the ranges of variations of K and α for which the number of detected modal regions remains constant, is sufficiently large. That demonstrates the robustness of the algorithm with respect to the discretization of the data space and to the adjustment of the hyperbolic operator parameters.

5. Results and discussion

Example of a synthetic color image

Let the synthetic color image with $256 * 256$ size coded on 24 bits (cf. figure 8). The sample of observations used as application of the proposed mode extraction approach to color image classification is constituted by 65536 color image pixels. The attributes of these observations are the 3 color components of the pixels taken in the color system representation. Each color pixel $P(i,j)$, where i and j are the spatial coordinates in the color image, can be represented by three levels (c_1, c_2, c_3) according to the considered color representation system. Figure 9 shows the sample of observations corresponding to the considered image taken in the color system representation RGB (Red, Green, Blue). Note that in statistical methods of segmenting, a cluster which may be valid in the feature space may not generally correspond to meaningful regions when it is mapped back in the spatial domain. Since, the effectiveness of the segmentation depends on the color reference system used in regard to the considered image, as well as on the algorithm.

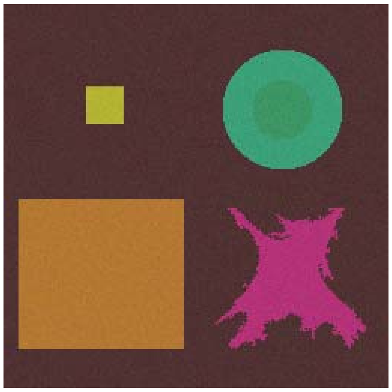

Figure 8: Original color image

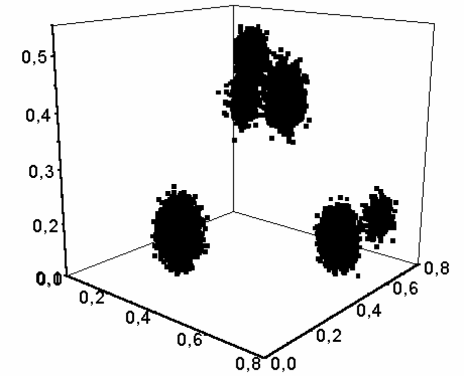

Figure 9: Pixels in the RGB color space

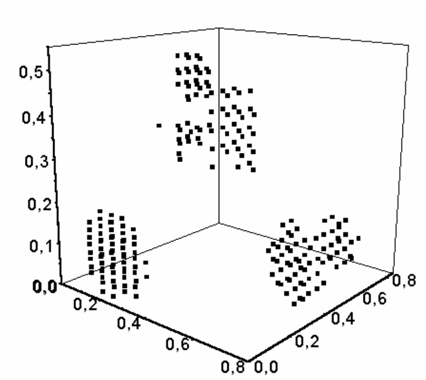

Figure 10: Prototypes in the RGB color space

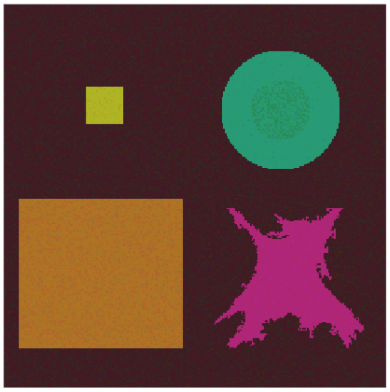

Figure 11: Segmented color image

For images treated in this paper, the use of RGB color space gives good results, but the analyst has to choose the color representation system [24] adapted to the context and complexity of each image.

Six modal regions, corresponding to the six homogeneous regions in the image, have been well extracted as connected components by the proposed mode extraction technique for $K=15$ and $\alpha=1.8$, according to the tuning algorithm procedure.

Figure 10 represents the observations falling into these different connected components corresponding to the identified modal regions of the underlying pdf. These observations are reconstructed in the image and are used as prototypes of the color regions present in the image.

The labeling procedure consists in assigning each pixel $P(i,j)$ whose color features are (c_1,c_2,c_3) to the region whose Euclidean distance with its corresponding mode prototypes is lowest. The color features of a labeled pixel are equal to coordinates of the associated mode prototype. This assignment rule is iterated until all the color pixels are assigned (cf. figure 11).

Example of a real color image

To illustrate the performances of the proposed approach on color real images, we try to apply it on the one represented in figure 13, which contains five kinds of homogeneous regions: four kinds of colored balls and the background. However the difficulty of the treatment of this image where the various clusters present an overlapping degree in the color space RGB (cf. figure 12), five modal regions have been detected by the proposed technique and have been used to have the segmented image, shown in figure 15.

Figure 12: Original image

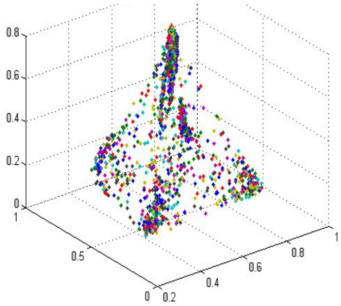

Figure 13: Pixels in the RGB space

Figure 14: Segmented image

	Original image	Image segmented by the competitive learning scheme	Image segmented by the proposed technique
Number of Black balls	24	34	24
Number of Green balls	17	7	17
Number of Red balls	20	20,5	20
Number of Orange balls	21	20,5	21

Table 1. Results of constructed balls by two approaches.

As it is difficult to compute a pixel confusion matrix from this image, we present a table (table 1) which indicates the number of the different-color balls which are reconstructed by the proposed procedure, based on the mode detection upon convexity dependant hyperbolic filter, and the segmentation scheme which uses the competitive learning technique developed in reference [1]. The criterion to decide whether a ball is well reconstructed is only a visual criterion.

6. Conclusion

A new unsupervised color image segmentation algorithm has been introduced, which is based on an original mode detection technique. The color of each pixel is represented, as an observation, by three levels according to the considered color representation system where the underlying probability density function is estimated by a non parametric technique.

The proposed algorithm makes the convexity dependant a separable recursive hyperbolic filter suitable for mode detection of the underlying pdf. It has been shown how the Laplacien, based on this hyperbolic operator, can be computed on a multivariate function. Under the assumption that there exists a one-to-one correspondence between the modes and the concave regions of the density function, and according to the characteristic theorem of convexity, modes of the pdf have been assimilated to regions where the hyperbolic Laplacien of the pdf is found negative. The observations falling into the different extracted modal regions of the underlying pdf identify the prototypes. Each pixel of the color image is finally affected to the region associated with the appropriate prototypes by means of an improved assignation rule.

On account of the filtering characteristic of the hyperbolic Laplacien and to its dependence on two parameters, an adequate tuning algorithm has been proposed which govern the behavior of the procedure.

A comparison between this segmentation method and a more classical color segmentation technique has shown the efficiency of the proposed procedure.

7. References

[1] T. Uchiyama and M. A. Arbib, Color image segmentation using competitive learning, I.E.E.E. Transactions on Pattern Analysis and Machine Intelligence, Vol. 16, n°12, 1994, pp. 1197-1206.

[2] A. Gillet, C. Botte-Lecocq, L. Macaire and J.-G. Postaire, Application of Fuzzy Mathematical Morphology for Unsupervised Color Pixels Classification, ifcs'2000, pp 69-75 , July 11-14, 2000, Namur, Belgium.

[3] A. Verikas, K. Malmqvist, L. Bergman, Color image segmentation by modular neural network, Pattern Recognition Letters, 1996.

[4] Devijver, P.A. and Kittler, K., Pattern recognition: A statistical approach, Englewood Cliff, NJ, Prentice-Hall international, 1982.

[5] Mizoguchi, R. and Shimura,S., Nonparametric Learning without a Teacher based on Mode Estimation, IEEE Trans. Comput., C-25 (11), 1976, pp.1109-1117

[6] A. Touzani, J. -G. Postaire, Mode detection by relaxation, IEEE Trans. Pattern Anal. and Machine Intell., Vol. 10 (6), 1988, pp. 970-978.

[7] A. Touzani, J. -G. Postaire, Clustering by mode boundary detection, Pattern Recognition Letters, Vol.9 (1), 1989, pp. 1-12.

[8] J. -G. Postaire, R. -D. Zhang, C. Botte-Lecocq, Cluster Analysis by Binary Morphology, IEEE. Trans. Pattern Anal. and Machine Intell., Vol. 15 (2), 1993, pp. 170-180.

[9] R. Touahni, A. Sbihi, M. J. Idrissi, Neural Morphological Approach for Pattern Classification. Intelligent Data Analysis, Vol. 5, 2001, pp. 263-282.

[10] N. P. Pal, J. C. Bezdeck, E. C. Tsao, Generalized Clustering Networks and Kohonen's self Organizing Schemes. IEEE Trans. Neural Network, Vol4, 1993, pp. 549-557.

[11] C.-P.-A. Vasseur and J.–G. Postaire, A Convexity Testing Method for Cluster Analysis, IEEE Trans. Syst. Man. Cybern. SMC-10 (3), 1980, pp. 145-149.

[12] R.-D. Zhang and J.-G. Postaire, Convexity Dependent Morphological Transformations for Mode Detection in Cluster Analysis, Pattern Recognition Letters, Vol. 27, n°1, 1994, pp. 135-148.

[13] J.-G. Postaire and S. Olejnik, A Relaxation Scheme for Improving a Convexity based Clustering Method, Pattern Recognition Letters, n°15, 1994, pp. 1211-1221.

[14] T. Rockafellar, Convex Analysis, Princeton Univ. Press, Princeton, W. J. 1970.

[15] Ph. Paillou, Detecting Step Edges in Noisy SAR Images: A New Linear Operator, IEEE Transactions on Geoscience and remote Sensing, Vol. 35, n°1, 1997, pp. 191-196.

[16] Y.-F. Wan, J.-G. Postaire and F. Cabestaing, A New Fast Recursive Algorithm for Image Boundary Detection, Software, Vol. 19, n°3, 1995, pp. 14-20.

[17] S.-D. Ma and B.-C. Li, Derivative Commutating by Multistage Filter, In Europe China Workshop on Geometric Modifying and Invariants, Xi'an, China, 1995.

[18] J. Canny, A Computational Approach to Edge Detection, IEEE Transactions on Pattern Analysis and Machine Intelligence. Vol. PAMI-8, n° 11, 1986, pp. 679-698.

[19] S. Sarkar and K. Boyer, Optimal Infinite Impulse Response Edge Detection Filters, IEEE Transactions on Pattern Analysis and Machine Intelligence. Vol. PAMI-13, n°11, 1991, pp. 1154-1171.

[20] J.-G. Postaire and C.-P.-A. Vasseur, A Fast Algorithm for Nonparametric Probability Density Estimation, IEEE Trans. Patt. Anal. & Machine intel. Vol. PAMI-4, n°6, 1982, pp. 663-666.

[21] T.-M. Cover and P.-E. Hart, Nearest Neighbor Pattern Classification, IEEE Trans. Info. Theory, Vol. IT-13, n°1, 1967, pp. 21-27.

[22] R.-O. Duda and P.-E. Hart, Pattern Classification and Scene Analysis, New York: Wiley, 1973.

[23] J.-G. Postaire and C.-P.-A. Vasseur, An Approximate Solution to Normal Mixture Identification with

Application to Unsupervised Pattern Classification, IEEE Trans. Pattern Anal. Machine Intell, Vol. PAMI-3, n°2, 1981, pp. 163-179.

[24] G. Wyszecky and W. S. Stiles, Color Science: Concepts and Methods, Quantitative Data and Formulae. Wiley, New York 1982.